Find all real solutions.

1. $3x^2 + 25x = -28$

$3x^2 + 25x + 28 = 0$

$(3x+21)(3x+4) = 0$

$(x+7)(3x+4) = 0$

$\boxed{x = -7, -4/3}$

2. $27x^3 = 64$

$27x^3 - 64 = 0$

$(3x-4)(9x^2 + 12x + 16)$

$\boxed{x = 4/3}$

$\dfrac{-12 \pm \sqrt{12^2 - 4(9)(16)}}{2(9)}$ not real

3. $x^3 - 3x^2 = 4x - 12$

$(x^3 - 3x^2) - 4x + 12 = 0$

$x^2(x-3) - 4(x-3) = 0$

$(x^2 - 4)(x-3) = 0$

$(x+2)(x-2)(x-3) = 0$

$\boxed{x = \pm 2, 3}$

4. $x^3y - 4xy = 0$

$xy(x^2 - 4) = 0$

$x = 0, \pm 2$

$y = 0$

5. $(a^2 + 1)^2 - 12(a^2 + 1) = -20$

$u = a^2 + 1$

$u^2 - 12u + 20 = 0$

$(u-10)(u-2) = 0$

$(a^2 + 1 - 10)(a^2 + 1 - 2) = 0$

$(a^2 - 9)(a^2 - 1) = 0$

$\boxed{a = \pm 3, \pm 1}$

6. $x^{\frac{1}{2}} + 3x^{\frac{-1}{2}} = 10x^{\frac{-3}{2}}$

$x^{1/2} + 3x^{-1/2} - 10x^{-3/2} = 0$

$x^{-3/2}(x^2 + 3x - 10) = 0$

$x^{-3/2}(x+5)(x-2) = 0$

$x = -5, 2, 0$

7. $x + 5 = 14 - \tfrac{1}{2}x$

$-9 = -\frac{3}{2}x$

$-9 \cdot \frac{2}{3} = x$

$\boxed{x = 6}$

8. $\dfrac{x}{x+1} - \dfrac{3}{x(x+1)} = \dfrac{2x-3}{x}$

$x^2 - 3 = (2x-3)(x+1)$

$x^2 - 3 = 2x^2 + 2x - 3x - 3$

$0 = x^2 - 1x$

$0 = x(x-1)$

$x = 0 \boxed{1}$

9. $x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$\boxed{x = 4, -3}$

10. $x^4 - 3x^2 + 2 = 0$

$x^2 = \dfrac{3 \pm \sqrt{(-3)^2 - 4(1)(2)}}{2(1)}$

$x^2 = \dfrac{3 \pm \sqrt{9 - 8}}{2}$

$x^2 = \dfrac{3 \pm 1}{2}$ $x^2 = 2, 1$

$\boxed{x = \pm\sqrt{2}, \pm 1}$

11. $3|x - 4| = 10$

$|x - 4| = \dfrac{10}{3}$

$x - 4 = \dfrac{10}{3}$ $\qquad x - 4 = -\dfrac{10}{3}$

$x = 2\tfrac{2}{3}$ $\qquad x = \dfrac{2}{3}$

12. $2x = 2 - \sqrt{6-x}$

$2x - 2 = -\sqrt{6-x}$

$(2x-2)^2 = 6 - x$

$4x^2 - 8x + 4 = 6 - x$

$4x^2 - 7x - 2 = 0$

$(4x+1)(x-2) = 0$

$4x = -1$ $\quad \cancel{x = 2}$

$x = -1/4$

Word Problems.

12. Find three consecutive integers whose sum is 360.

$$n + n+1 + n+2 = 360$$
$$3n + 3 = 360$$
$$3n = 357$$
$$n = \boxed{119, 120, 121}$$

13. If Ben invests $3000 at 4% interest per year, how much additional money must he invest at 5.5% annual interest to ensure that the interest he receives each year is 4.5% of the total amount invested?

$$.04(3,000) + .055x = .045(3000 + x)$$
$$120 + .055x = 135 + .045x$$
$$.01x = 15$$
$$\boxed{x = 1500}$$

14. A rectangular parcel of land is 70 ft longer than it is wide. Each diagonal between opposite corners is 130 ft. What are the dimensions of the parcel?
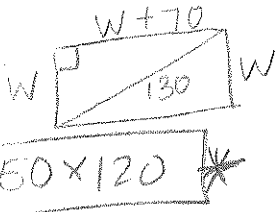
$$W^2 + (W+70)^2 = 130^2$$
$$W^2 + W^2 + 140W + 4900 = 16900$$
$$2W^2 + 140W - 12000 = 0$$
$$2(W^2 + 70W - 6000) = 0$$
$$x = \frac{-70 \pm \sqrt{70^2 - 4(1)(-6000)}}{2(1)} = \frac{-70 \pm \sqrt{28900}}{2} = \frac{-70 + 170}{2} \quad \boxed{x = 50}$$

50 × 120

15. A large plywood box has a volume of 180 ft³. Its length is 9 ft greater than its height, and its width is 4 ft less than its height. What is the height of the box?

$$h(h+9)(h-4) = 180$$
$$h^3 + 5h^2 - 36h = 180$$
$$h^3 + 5h^2 - 36h - 180 = 0$$
$$h^2(h+5) - 36(h+5) = 0$$
$$(h+6)(h-6)(h+5) = 0$$

$$\boxed{\begin{array}{l} h = 6 \\ L = 15 \\ W = 2 \end{array}}$$

Solve each inequality. Write the answer using interval notation, and sketch the solution on the real number line.

16. $-4 < 5 - 3x \le 17$

$$-9 < -3x \le 12$$
$$3 > x \ge -4$$

$[-4, 3)$

17. $x^3 + x^2 > 2x$

$$x^3 + x^2 - 2x > 0$$
$$x(x^2 + x - 2) > 0$$
$$x(x-1)(x+2) > 0$$
$$x = 0, 1, -2$$

$(-2, 0) \cup (1, \infty)$

18. $|x - 4| < 3$

$$x - 4 < 3$$
$$x - 4 > 3$$
$$x < 7, \quad x > 7 + 1$$

$(7+1, 7)$

19. $\dfrac{2x - 3}{x + 1} \le 1$

$$\frac{2x-3}{x+1} - \frac{x+1}{x+1} \le 0$$
$$\frac{2x - 3 - x - 1}{x+1} \le 0$$
$$\frac{x - 4}{x + 1} \le 0$$

Describe and sketch the regions given by each set.
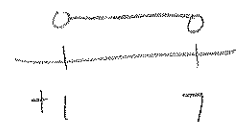
Find the center and radius of each circle.

26. $x^2 + y^2 = 25$.

(0,0)   r=5

center: (0,0)
r: 5

27. $(x - 2)^2 + (y + 1)^2 = 9$

center: (2,-1)
radius: 3

28. $x^2 + 6x + y^2 - 2y + 6 = 0$

$x^2 + 6x + 9 + y^2 - 2y + 1 = -6 + 9 + 1$

$(x+3)^2 + (y-1)^2 = 4$

center: (-3,1)
radius: 2

Find the equation of the circle.

29. Centered at (-2,2); passes through (0,2)

$(0+2)^2 + (2-2)^2 = r^2$

$4 + 0 = r^2$

$r = 2$

$(x+2)^2 + (y-2)^2 = 4$

30. Centered at (-1,1); passes through (2,0)

$(2+1)^2 + (0-1)^2 = r^2$

$9 + 1 = r^2$

$10 = r^2$

$(x+1)^2 + (y-1)^2 = 10$

Find an equation for the line with the given property.

31. It passes through the point (3, -6) and is parallel to the line $3x + y - 10 = 0$.

$y = -3x + 10$

$-6 = -3(3) + b$

$-6 = -9 + b$

$3 = b$

$y = -3x + 3$

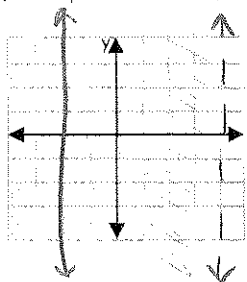32. It has x-intercept 6 and y-intercept 4.

(6,0); (0,4)

$\frac{4-0}{0-6} = \frac{4}{-6} = -\frac{2}{3}$

$y - 0 = -\frac{2}{3}(x-6)$

$y = -\frac{2}{3}x + 4$

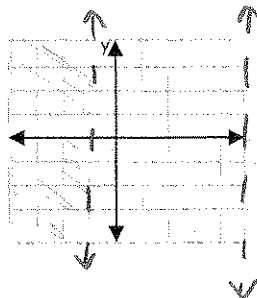Describe and sketch the regions given by each set.

20. $\{(x,y) \mid -2 \le x < 4\}$

21. $\{(x,y) \mid |x-2| > 3\}$

$x - 2 > 3$ or $x - 2 < -3$
$x > 5$ $x < -1$

23. Plot the points P(0, 3), Q(3, 0), and R(6, 3) in the coordinate plane.

a) Where must the point S be located so that PQRS is a square? $(6, 6)$

b) Find the area of PQRS

$PQ = \sqrt{3^2 + 3^2} = \sqrt{18}$

$QR = \sqrt{18}$

$area = \boxed{18 \text{ units}^2}$

24. Let P(-3, 1) and Q(5, 6) be two points in the coordinate plane.

Plot P and Q in the coordinate plane.

Find the distance between P and Q. $\sqrt{8^2 + 5^2} = \sqrt{89}$

Find the midpoint of the segment PQ. $\left(\frac{-3+5}{2}, \frac{1+6}{2}\right) = 1, 7/2$

Find the slope of the line that contains P and Q.

$\frac{6-1}{5+3} = 5/8$

Find the perpendicular bisector of the line that contains P and Q. pt $(1, 7/2)$ $m = -8/5$ $y = \frac{-8}{5} + \frac{51}{10}$

Find an equation for the circle for which the segment PQ is the diameter. $c = 1, 7/2$

$r = \frac{\sqrt{89}}{2}$

$(x-1)^2 + (y - 7/2)^2 = \frac{89}{4}$

25. Find the x and y intercepts of the graph of

a) $y = x^2 - 4$.

x-int: set y=0   y-int:
$0 = x^2 - 4$ $y = 0 - 4$
$4 = x^2$
$\pm 2 = x$

$\boxed{(-2, 0)}$ $\boxed{(0, -4)}$
$\boxed{(2, 0)}$

b) $y = \frac{2x}{x+1}$

x-int: y-int:
$0 = \frac{2x}{x+1}$ $y = \frac{2(0)}{0+1} = 0$

$(0, 0)$ $\boxed{(0, 0)}$